

AE-803

M.A./M.Sc. (Previous)
Term End Examination, 2016-17

MATHEMATICS

Optional

Paper - II

Advanced Discrete Mathematics

Time : Three Hours] [Maximum Marks : 100
[Minimum Pass Marks : 36

Note : Answer any **five** questions. Answer to each question should begin on a fresh page. All questions carry equal marks.

1. (a) Show that $p \vee \sim (q \wedge r)$ is equivalent to $(p \vee \sim q) \vee \sim r$.
(b) State and prove De-Morgan law.
2. (a) Show that the following argument is valid :

$$\begin{array}{c} p \\ p \Rightarrow q \\ q \end{array}$$

(2)

- (b) Explain quantifiers in detail.
3. (a) Define semigroup and monoid.
- (b) Show that every finite semigroup has an idempotent element.
4. (a) Show that for any commutative monoid $(M, *)$, the set of idempotent elements of M forms a submonoid.
- (b) Let $(M, *)$, be a monoid, then show that there exists a subset $T \subseteq M$ such that $(M, *)$ is isomorphic to monoid $(T, *)$.
5. (a) Let (L, \leq) be a lattice in which $*$ and \oplus denote the operations of meet and join respectively. Then show that for any $a, b \in L$
- $$a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$$
- (b) Show that every chain is a distributive lattice.
6. (a) State and prove associative law in Boolean algebra.
- (b) Show that complement element of every element is unique in Boolean algebra.
7. (a) Show that the value of complete disjunctive normal form is one.

(3)

(b) In any Boolean algebra, show that

$$(a + b) (b + c) (c + a) = a.b + b.c + c.a$$

8. (a) Explain finite state machine in detail.

(b) Show that $L = \{a^k : k = i^2, i \geq 1\}$ is not a finite state language.

9. (a) Explain the following terms :

(i) Path

(ii) Circuit

(b) Show that a tree with n vertices has $n-1$ edges.

10. (a) State and prove Euler formula for connected graph.

(b) What do you mean by Konigsberg Bridge problem ?
